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LETTER TO THE EDITOR

Effective permeability of sandstone–shale reservoirs by a random walk method

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Abstract. A random walk algorithm for calculating the effective permeability of random mixtures of two finite permeability components (e.g. sandstone and shale) is introduced. It is an extension of the 'ant in a labyrinth' algorithm, which only applies when one of the components has zero permeability. Numerical results obtained from the proposed random walk algorithm are shown to be in close agreement with those obtained from the conventional finite difference method on three-dimensional cubic lattices. In contrast, it is shown that the hybrid random walk algorithms which have been used in the past to study the critical behaviour of two-component random conductor mixtures are inadequate for the problem under consideration. The proposed random walk algorithm provides a computationally efficient method for calculating the effective permeability and does not require the explicit solution of the fluid flow through the random mixture.

The flow simulation programs used in petroleum reservoir engineering contain the implicit assumption that the properties of a reservoir can be considered to be homogeneous on the scale of the grid blocks used in the finite difference scheme. At best this assumption is questionable and it is particularly unsatisfactory in the case where the reservoir rock, which is predominantly sandstone, contains shale lenses of an intermediate length which cannot be correlated between wells. The shale lenses have a very low permeability, a factor of three to seven orders of magnitude less than the sandstone permeability, and can drastically alter the flow characteristics of a reservoir. The problem of determining the effective permeability of a heterogeneous reservoir is of prime importance in assessing the economic viability of producing the reservoir and has been the subject of much recent research in the petroleum industry [1, 2].

In this paper we discuss the calculation of the effective permeability of sandstone-shale reservoirs by random walk techniques. In computing the effective permeability, κ_{eff}, of a random porous medium in numerical simulations, the usual procedure is to solve the flow equations by way of a finite difference scheme, to find the pressure distribution at the grid points and to equate the integrated flux over any cross section of the field with the flux corresponding to a field of uniform permeability κ_{eff}, in accordance with Darcy's law (a stochastic approach is used whereby ensemble averages are taken over a large number of realisations of the permeability distribution). However, if one is only interested in calculating the effective permeability it is not necessary to go to the trouble of solving the flow equations. On large lattices, it is more efficient to use a random walk method, familiar from work on diffusion and percolation theory [3].

A step in this direction was taken by Drummond *et al* [4]. They developed a discrete time numerical algorithm to integrate the Ito-type stochastic differential equation corresponding to the flow problem. In their first-order algorithm, the update step at time τ for a particle located at position x is given by

$$\Delta x = \nabla \kappa(x) \Delta \tau + (2\kappa(x)\Delta \tau)^{1/2} \eta \qquad (1)$$

where $\Delta\tau$ is the time step and η is a vector whose components are independent Gaussian random variables with zero mean and unit variance. Thus, it contains a local drift term which depends on the gradient of the permeability at x, and a local diffusion term, which contains a random factor. In the case in which the permeability is constant, the algorithm reduces to a pure random walk, i.e. Brownian motion. Drummond *et al* applied their methods to the effective permeability problem for continuous permeability distributions, such as the log-normal distribution, using small variances, so that comparisons with the results of perturbation theory could be made.

In reservoir simulations, the permeability is defined on discrete grid blocks and is often drawn from a discontinuous permeability distribution. For example, it has been shown that sandstone-shale reservoirs can be well approximated by using a bimodal distribution of the permeabilities, i.e. by using some mean value, κ_{ss}, for the sandstone permeability (usually the geometric mean) and another value, κ_{sh}, for the shale permeability [1]. This is because the permeability variation within each type of rock is negligible compared with the variation between rock types. In this case, it is evident from equation (1) that problems would arise in the algorithm of Drummond *et al* at the interface region between sandstone and shale where $\nabla\kappa$ is large. Even if the permeability distribution were smoothed out to make it continuous, very small time increments would have to be taken in order to prevent the spatial step size from blowing up. Hence, for a discrete permeability distribution it would be preferable to have a discrete lattice random walk algorithm. The algorithm proposed in this paper is limited in the sense that it only applies to the two-component permeability case. However, its structure suggests a possible extension to multicomponent systems and work is in progress on this.

The problem of calculating the effective permeability of media with a bimodal distribution of permeabilities is mathematically equivalent to calculating the effective conductivity of two-component random resistor networks [5]. It is well known that one can replace the conductivity problem with a diffusion problem using the Nernst-Einstein relation [6, 7] which states that the diffusivity is proportional to the conductivity. For a homogeneous medium the diffusivity can be calculated using discrete lattice random walks. Diffusivity is given by the slope of the line $\langle R^2(t)\rangle$ against t, at large times, where $\langle R^2(t)\rangle$ is the mean square distance travelled by the random walkers at time t.

Our random walk algorithm for the two-component permeability case involves a random walker, or 'ant', which performs a discrete lattice walk on each component separately, with the jump-rate slowed down by a factor of κ_{sh}/κ_{ss} in the shale region as compared with the sandstone region, and with an appropriate probability, the 'interface' probability, defined for jumping from one component to another when starting from an interface site. The interface probability corresponds to the drift term which appears in the stochastic differential equation (1) in regions where the permeability gradient is non-zero. The issue of what the interface probability should be and how the algorithm should be set up is discussed below. It is shown that previous related hybrid random walk algorithms [8, 9], although adequate for their intended purpose of studying the critical behaviour, do not give the correct value of the permeability at all volume fractions. This is because they use a 'myopic ant'-type algorithm [10] as distinct from the more impartial 'blind ant'-type algorithm which is proposed in this letter.

When the shale permeability is assumed to be zero, the interface probability must be such as to ensure that total reflection occurs from a shale interface back into the sandstone. In this case, one can use the 'ant in a labyrinth' algorithm of de Gennes [6], as was recently done by Schwartz and Banavar [11] in their calculations of the electrical conductivity and DC permeability of isotropic and anisotropic packings of multisize granular materials. In the 'ant in a labyrinth' algorithm, the ant randomly chooses one of its nearest neighbours and moves there if the chosen site has finite permeability (i.e. is in sandstone) or stays put if it has zero permeability (i.e. is in shale). In either case, the simulation time is incremented by one unit. This is an example of a 'blind ant' algorithm. A related 'myopic ant' algorithm is one in which the ant only chooses from those of its neighbours which have finite permeability and always moves to the selected site. It has been shown [12] that the blind ant is a more impartial sampler of a cluster because, asymptotically, it visits each site of a cluster with equal probability, while this is not true of the myopic ant. Hence, in general we would expect the two algorithms to give different answers, as is illustrated in figure 1. However, the myopic ant algorithm can still be used to study the critical behaviour of systems because it has been shown [12] that the two types of ant are in the same universality class, i.e. give rise to the same critical exponents.

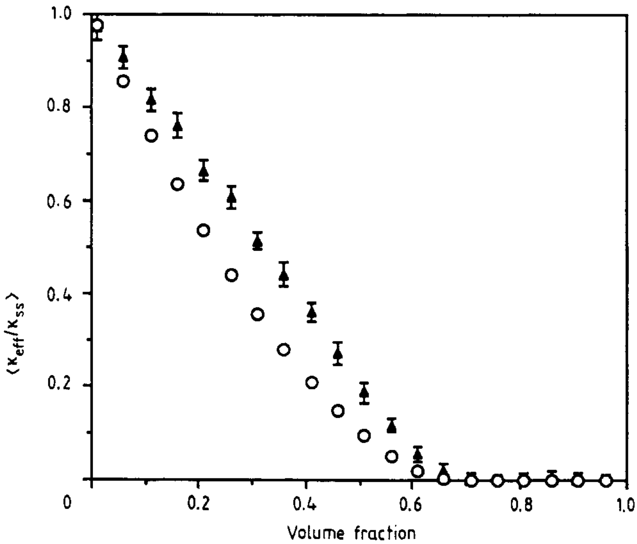

Figure 1. Effective permeability plotted against shale volume fraction for a random, uncorrelated, two-component system with zero shale permeability. Comparison between myopic (△) and blind (○) ant algorithms.

For the case in which the shale permeability is not zero, Bunde *et al* [8, 9] suggested an algorithm which they called 'the Boston termite 1', which will be referred to in this letter as 'the Boston ant', for consistency. In the Boston ant algorithm, the transition probability at a given site, Π_i, to the ith nearest-neighbour site is given by $\Pi_i = \kappa_i/\Sigma_j\kappa_j$, where $\kappa_i = (\kappa_{ss}$ or $\kappa_{sh})$ is the permeability at site i and the sum Σ_j extends over the nearest neighbours of site i. The ant's time step, τ_{sh} in shale and τ_{ss} in sandstone, is inversely proportional to the permeability, i.e. $\tau_{sh}/\tau_{ss} = \kappa_{ss}/\kappa_{sh}$. The time step is explicitly scaled so that the walker slows down by the appropriate amount in the shale region relative to the sandstone region. The total elapsed time, $t = N_{ss}\tau_{ss} + N_{sh}\tau_{sh}$, is related to the mean-square displacement and the diffusion constant by:

$$\langle R^2(t)\rangle \propto Dt \tag{2}$$

where N_{ss} and N_{sh} are the total number of steps in sandstone and shale respectively. Time ensemble averages are considered. In the limiting case $\kappa_{sh} = 0$, the Boston ant reduces to the previously discussed myopic ant algorithm—the ant moves within the sandstone, making a jump to a new site at each time step.

In the proposed new algorithm, the permeabilities are scaled so that $\kappa_{ss} = 1$ and $\kappa_{sh} = \kappa_{sh}/\kappa_{ss}$. The ant at site i chooses one of its nearest neighbours, say j, at random (hence, it is a blind ant) and moves there with a probability given by:

$$\Pi_{ij} = \frac{2\kappa_i\kappa_j}{\kappa_i + \kappa_j} \tag{3}$$

or stays put with probability $(1 - \Pi_{ij})$. In either case, the simulation time is incremented by one unit. In this algorithm, the same rules apply regardless of whether site i is in sandstone or shale or at an interface. It is not necessary to scale the time step explicitly since this is achieved implicitly by including the factor κ_i in the jump probability Π_{ij}. With the proposed algorithm it is possible to use the exact enumeration method [12] to enumerate all possible random walks, since the transfer matrix of jump probabilities Π_{ij} is well defined. (In practice, we did not use the exact enumeration method because we wanted to simulate long walks on large, three-dimensional lattices).

Consider how the algorithm operates in different regions:

(i) if i and j are both in sandstone, then $\Pi_{ij} = 1$, so the ant performs the usual discrete time random walk within a sandstone region;

(ii) if i and j are both in shale, then $\Pi_{ij} = \kappa_{sh}$, which gives the properly scaled random walk within a shale region;

(iii) if i and j are in different regions, then $\Pi_{ij} = 2\kappa_{sh}/(1 + \kappa_{sh})$.

It is a little surprising that we have chosen Π_{ij} to be symmetric. Intuitively, it is more probable to jump from a region of low permeability to one of high permeability than *vice versa*. However, the jump probability for going from shale to sandstone contains a slow-down factor κ_{sh}, because the ant moves more slowly in shale, and this has the effect of symmetrising the probabilities. Note that the algorithm reduces to he 'ant in a labyrinth' algorithm when $\kappa_{sh} = 0$ and to a simple lattice random walk in the limit in which $\kappa_{sh} = \kappa_{ss}$.

The jump probability Π_{ij} has been defined to be given by the harmonic mean of the permeabilities of neighbouring sites. This was suggested by the fact that for flow in one dimension, or for flow perpendicular to the layers in a layered medium, the effective permeability is given by the harmonic mean of the constituent permeabilities. We speculate that the proposed algorithm could be directly extended to the multicomponent permeability case, with the maximum site permeability normalised to one. Work is in progress to verify this.

We have compared the results of the proposed algorithm with those obtained from a finite difference algorithm and from the Boston ant algorithm of Bunde *et al* for a random, two-component medium with varying shale volume fraction and with values for the permeability contrast ratio, $\alpha = \kappa_{sh}/\kappa_{ss}$, ranging from 0 to 1. Averages were taken over 10 realisations of the permeability distribution on a 30^3 lattice, with 1000 random walkers taking 10 000 steps on each realisation. Results are shown in figure 2. Clearly the results of the proposed algorithm are completely consistent with the results of the finite difference algorithm in all cases, whereas those of the Boston ant algorithm are not. The different algorithms give results which converge as $\alpha \to 1$, as is to be expected, since they are equivalent when $\alpha = 1$ (i.e. when there is constant permeability throughout the medium).

As illustrated by Schwartz and Banavar [11], random walk algorithms can be used to study anisotropic systems as well as isotropic ones. In figure 3 we show the results which we obtained in numerical simulations on an anisotropic system with a vertical to horizontal anisotropy ratio of 1 : 10. The permeability contrast ratio is also 1 : 10. The vertical effective permeabilities obtained using the random walk algorithm are completely consistent with those obtained using a finite difference algorithm. Averages were taken over 10 realisations of the permeability distribution on a 30^3 lattice. The permeability distributions were generated by an indicator random function technique as described by Journel and Huijbregts [13]. In this technique, first- and second-order moments of a spatial random function describing the presence or absence of shale are used to characterise shale volume fraction and spatial continuity. Figure 4 shows a

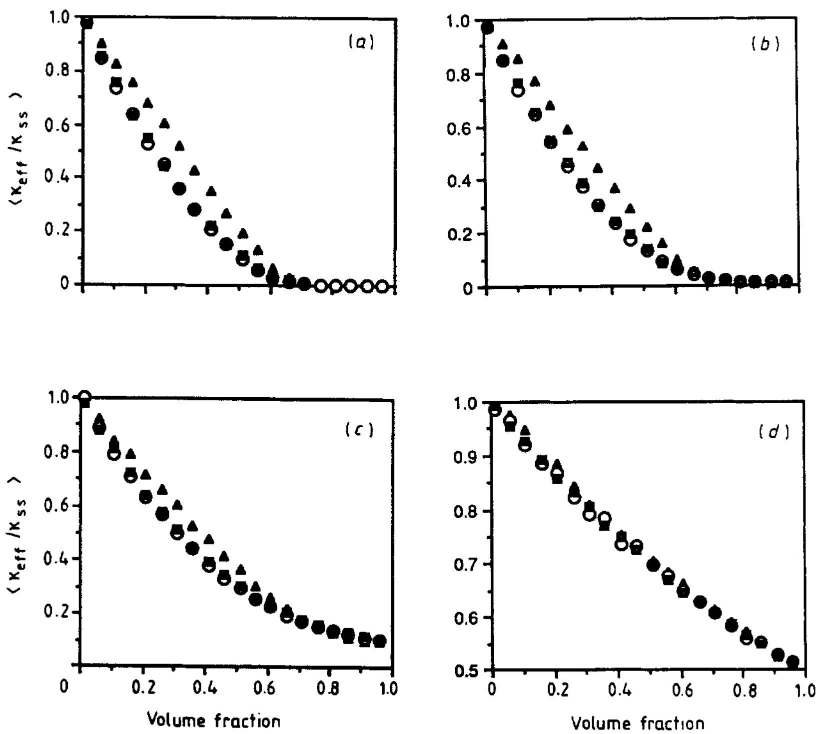

Figure 2. Effective permeability plotted against shale volume fraction for a random, uncorrelated, two-component system with different permeability contrast ratios, $\alpha = \kappa_{sh}/\kappa_{ss}$. (*a*) $\alpha = 0.001$, (*b*) $\alpha = 0.01$, (*c*) $\alpha = 0.1$ and (*d*) $\alpha = 0.5$. Comparison between the Boston ant algorithm (△), the proposed algorithm (○), and the finite difference algorithm (□). In all cases, the error bars are of the order of twice the size of the plotting symbols used and have been omitted for clarity.

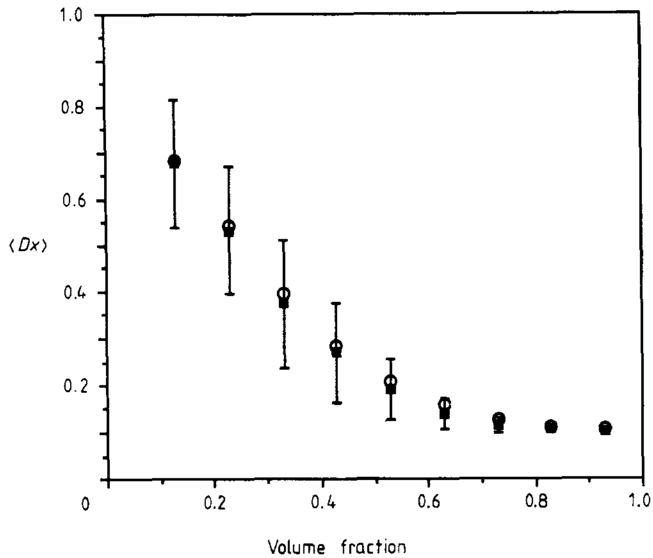

Figure 3. Effective permeability plotted against shale volume fraction measured perpendicular to the plane of the shale lenses in a system with a vertical to horizontal anisotropy ratio of 1 : 10. The permeability contrast ratio is also 1 : 10. Comparison between random walk algorithm (○) and finite difference algorithm (□).

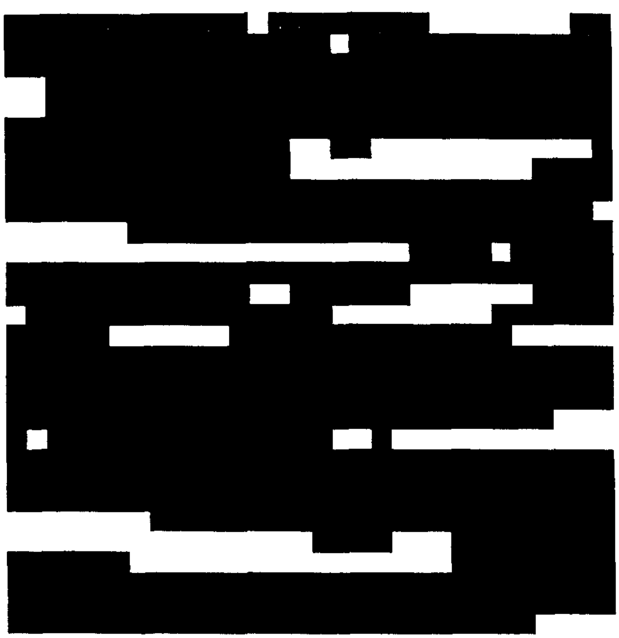

Figure 4. Cross-section of a realisation of a sandstone–shale permeability distribution on a 30^3 lattice with a shale volume fraction of 0.25 and a vertical to horizontal anisotropy ratio of 1 : 10.

cross section of one such realisation of a sandstone–shale permeability distribution on a lattice of size 30^3, with a shale volume fraction of 0.25 and with a vertical to horizontal anisotropy ratio of 1 : 10.

In conclusion, a random walk algorithm has been developed to calculate the effective permeability of random mixtures of two finite permeability components. It is intended to use this algorithm to study the dependence of the effective permeability of sandstone–shale reservoirs on factors such as the degree of anisotropy, the shape of shale inclusions and the correlations between sandstone and shale.

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